

## Basic Identities of Boolean Algebra

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1.	$X + 0 = X$	2.	$X \cdot 1 = X$	
3.	$X + 1 = 1$	4.	$X \cdot 0 = 0$	
5.	$X + X = X$	6.	$X \cdot X = X$	
7.	$X + \bar{X} = 1$	8.	$X \cdot \bar{X} = 0$	
9.	$\overline{\bar{X}} = X$			
<hr/>				
10.	$X + Y = Y + X$	11.	$XY = YX$	Commutative
12.	$X + (Y + Z) = (X + Y) + Z$	13.	$X(YZ) = (XY)Z$	Associative
14.	$X(Y + Z) = XY + XZ$	15.	$X + YZ = (X + Y)(X + Z)$	Distributive
16.	$\overline{X + Y} = \bar{X} \cdot \bar{Y}$	17.	$\overline{X \cdot Y} = \bar{X} + \bar{Y}$	DeMorgan's

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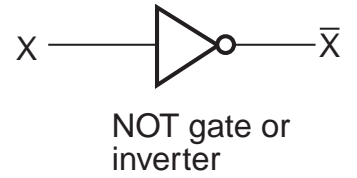
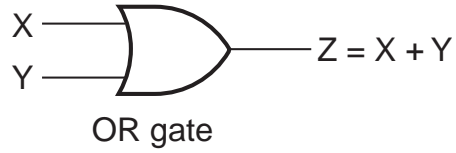
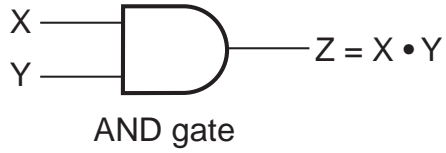
## Minterms for Three Variables

X	Y	Z	Product Term	Symbol	m <sub>0</sub>	m <sub>1</sub>	m <sub>2</sub>	m <sub>3</sub>	m <sub>4</sub>	m <sub>5</sub>	m <sub>6</sub>	m <sub>7</sub>
0	0	0	$\overline{X}\overline{Y}\overline{Z}$	m <sub>0</sub>	1	0	0	0	0	0	0	0
0	0	1	$\overline{X}\overline{Y}Z$	m <sub>1</sub>	0	1	0	0	0	0	0	0
0	1	0	$\overline{X}Y\overline{Z}$	m <sub>2</sub>	0	0	1	0	0	0	0	0
0	1	1	$\overline{X}YZ$	m <sub>3</sub>	0	0	0	1	0	0	0	0
1	0	0	$X\overline{Y}\overline{Z}$	m <sub>4</sub>	0	0	0	0	1	0	0	0
1	0	1	$X\overline{Y}Z$	m <sub>5</sub>	0	0	0	0	0	1	0	0
1	1	0	$XY\overline{Z}$	m <sub>6</sub>	0	0	0	0	0	0	1	0
1	1	1	$XYZ$	m <sub>7</sub>	0	0	0	0	0	0	0	1

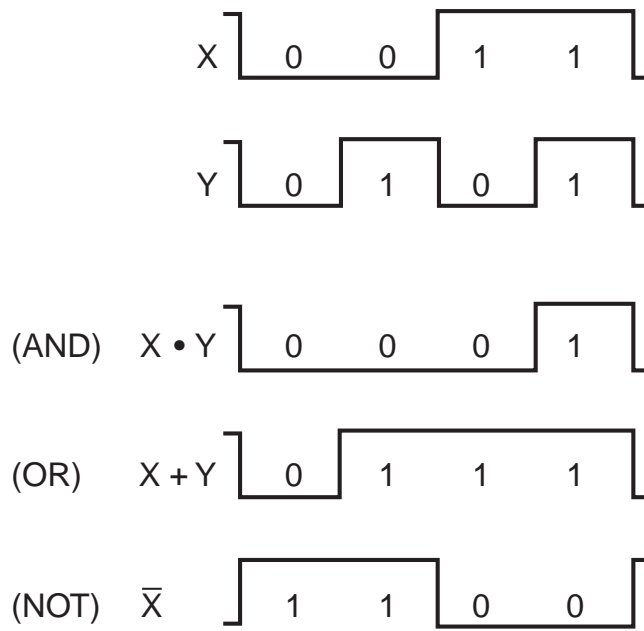
## Maxterms for Three Variables

X	Y	Z	Sum Term	Symbol	M <sub>0</sub>	M <sub>1</sub>	M <sub>2</sub>	M <sub>3</sub>	M <sub>4</sub>	M <sub>5</sub>	M <sub>6</sub>	M <sub>7</sub>
0	0	0	$X + Y + Z$	M <sub>0</sub>	0	1	1	1	1	1	1	1
0	0	1	$X + Y + \bar{Z}$	M <sub>1</sub>	1	0	1	1	1	1	1	1
0	1	0	$X + \bar{Y} + Z$	M <sub>2</sub>	1	1	0	1	1	1	1	1
0	1	1	$X + \bar{Y} + \bar{Z}$	M <sub>3</sub>	1	1	1	0	1	1	1	1
1	0	0	$\bar{X} + Y + Z$	M <sub>4</sub>	1	1	1	1	0	1	1	1
1	0	1	$\bar{X} + Y + \bar{Z}$	M <sub>5</sub>	1	1	1	1	1	0	1	1
1	1	0	$\bar{X} + \bar{Y} + Z$	M <sub>6</sub>	1	1	1	1	1	1	0	1
1	1	1	$\bar{X} + \bar{Y} + \bar{Z}$	M <sub>7</sub>	1	1	1	1	1	1	1	0

Digital Logic Gates

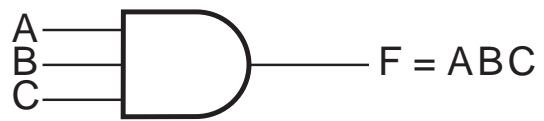


(a) Graphic symbols

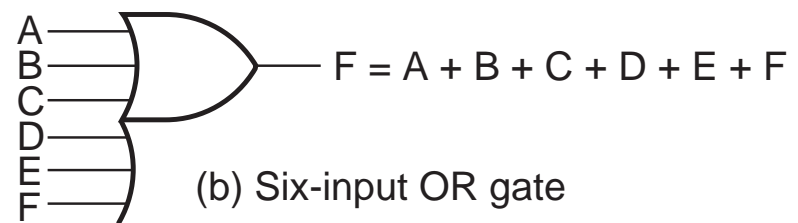


(b) Timing diagram

## Gates with More than Two Inputs

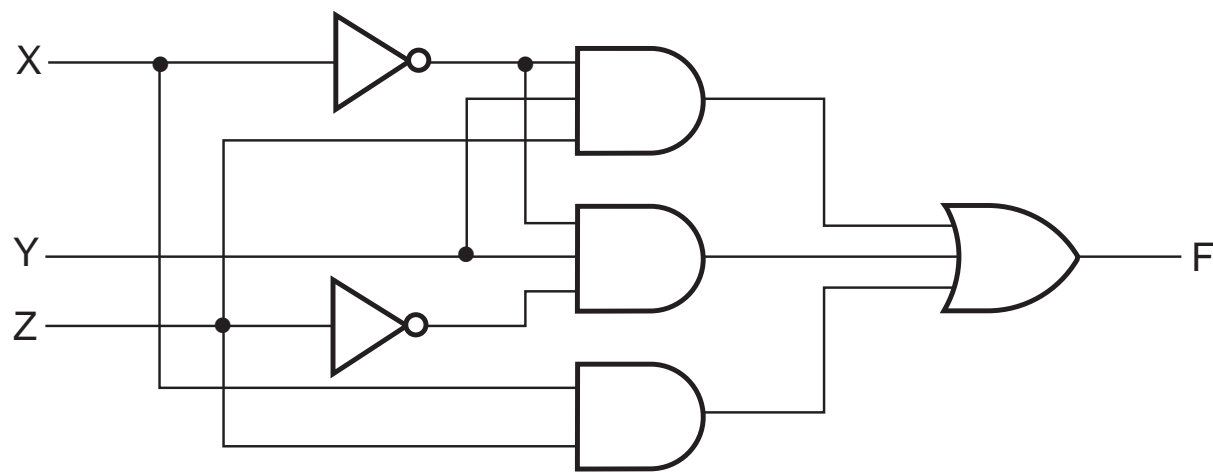


(a) Three-input AND gate

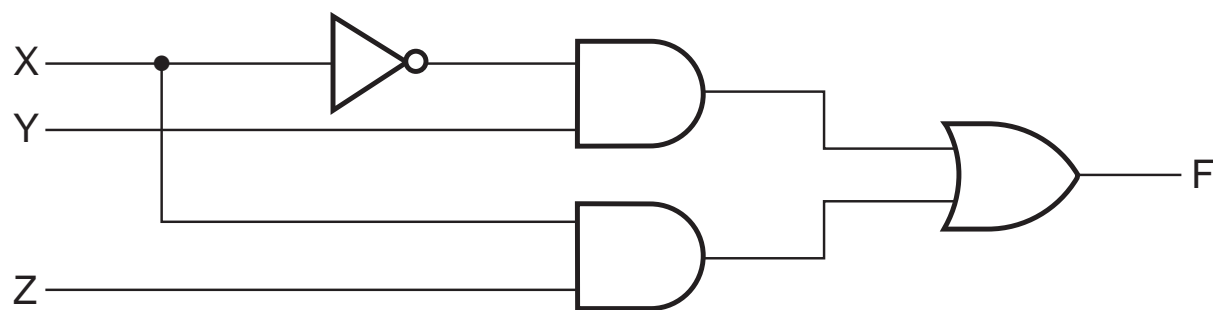


(b) Six-input OR gate

## Implementation of Boolean Function with Gates

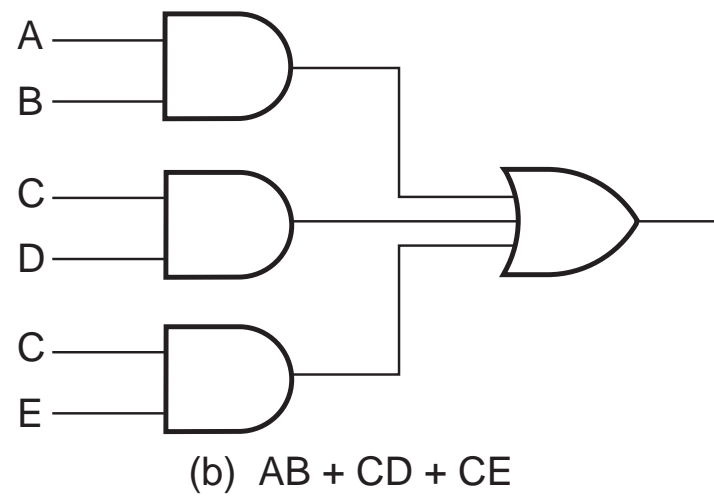
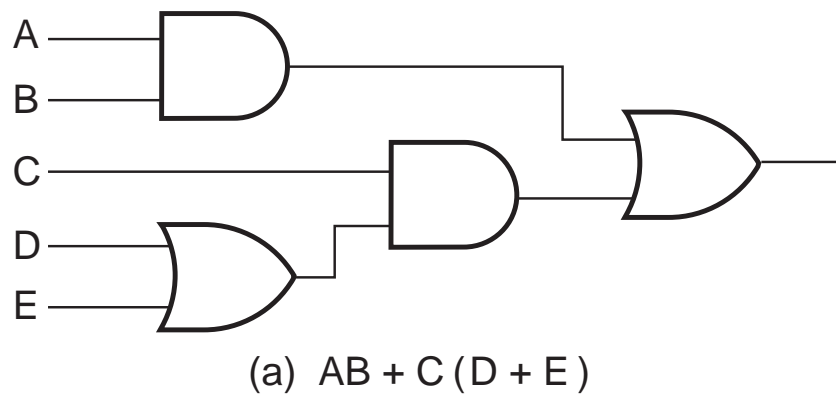


$$(a) F = \bar{X}YZ + \bar{X}Y\bar{Z} + XZ$$



$$(b) F = \bar{X}Y + XZ$$

## Three-Level and Two-Level Implementation



## Two-Variable Map

$m_0$	$m_1$
$m_2$	$m_3$

(a)

		0	1
Y X			
0	$\bar{X}\bar{Y}$	$\bar{X}Y$	
1	$X\bar{Y}$	$XY$	

(b)



## Three-Variable Map

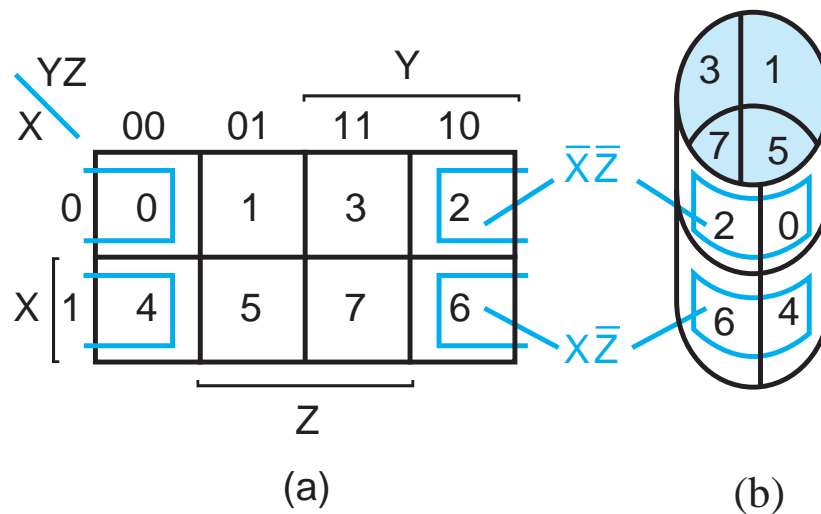
$m_0$	$m_1$	$m_3$	$m_2$
$m_4$	$m_5$	$m_7$	$m_6$

(a)

		YZ			
				Y	
	X	00	01	11	10
	0	$\bar{X}\bar{Y}\bar{Z}$	$\bar{X}\bar{Y}Z$	$\bar{X}YZ$	$\bar{X}Y\bar{Z}$
	1	$X\bar{Y}\bar{Z}$	$X\bar{Y}Z$	$XYZ$	$XY\bar{Z}$
		Z			

(b)

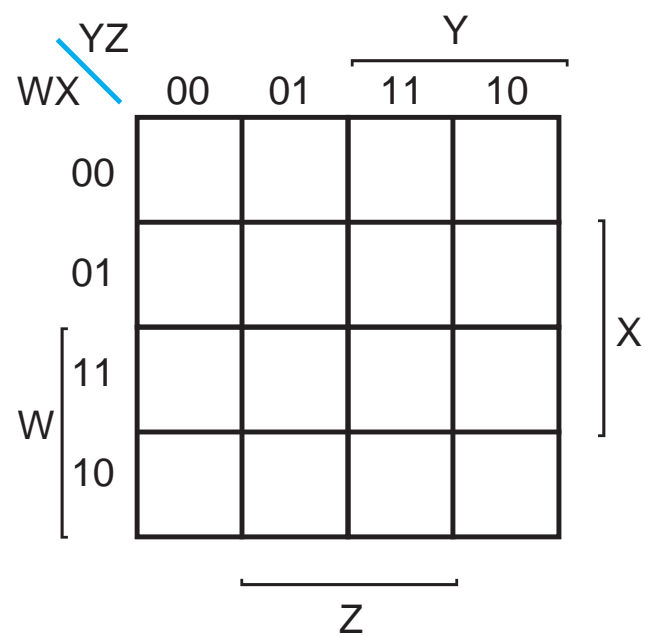
## Three-Variable Map: Flat and on a Cylinder to Show Adjacent Squares



## Four-Variable Map

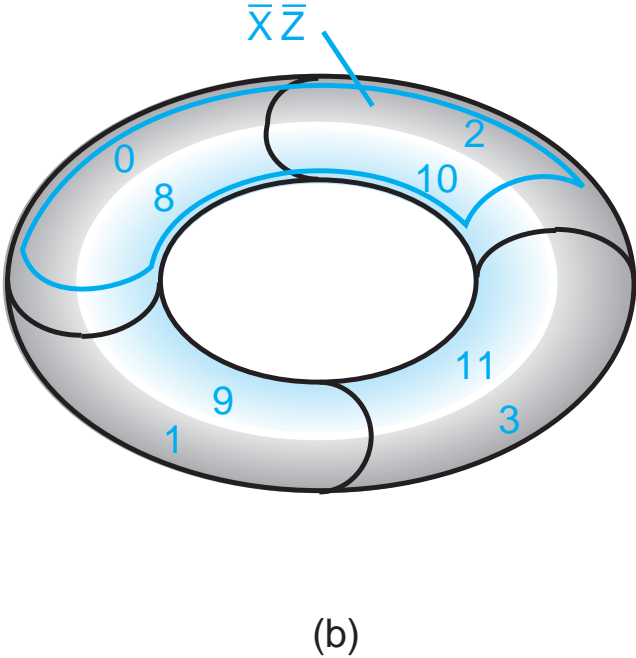
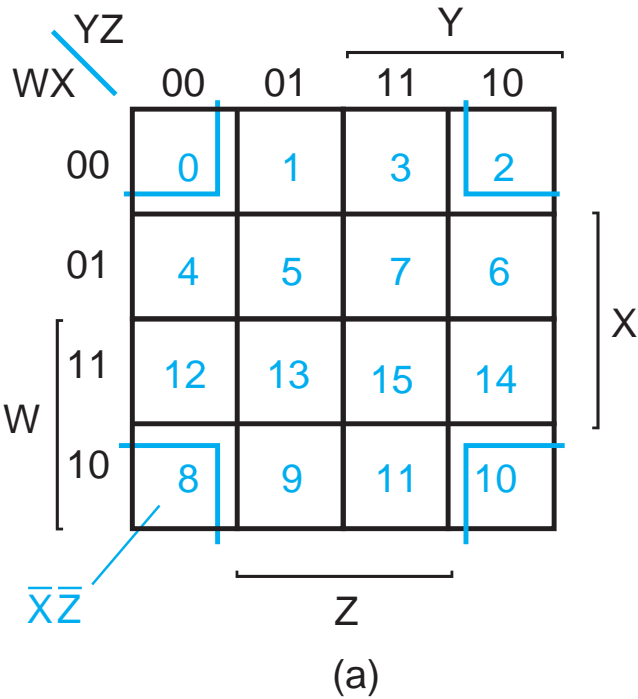
$m_0$	$m_1$	$m_3$	$m_2$
$m_4$	$m_5$	$m_7$	$m_6$
$m_{12}$	$m_{13}$	$m_{15}$	$m_{14}$
$m_8$	$m_9$	$m_{11}$	$m_{10}$

(a)


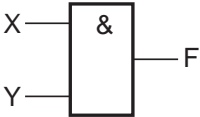
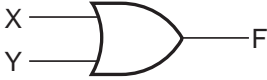
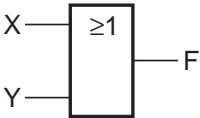
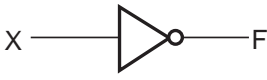
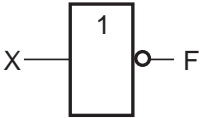
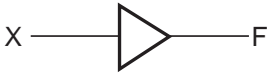
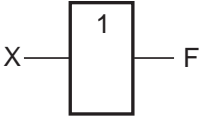

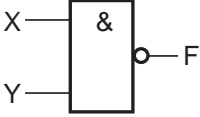

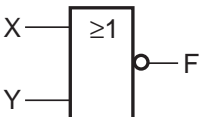

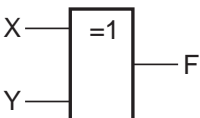

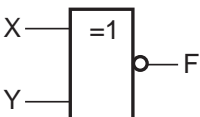


(b)

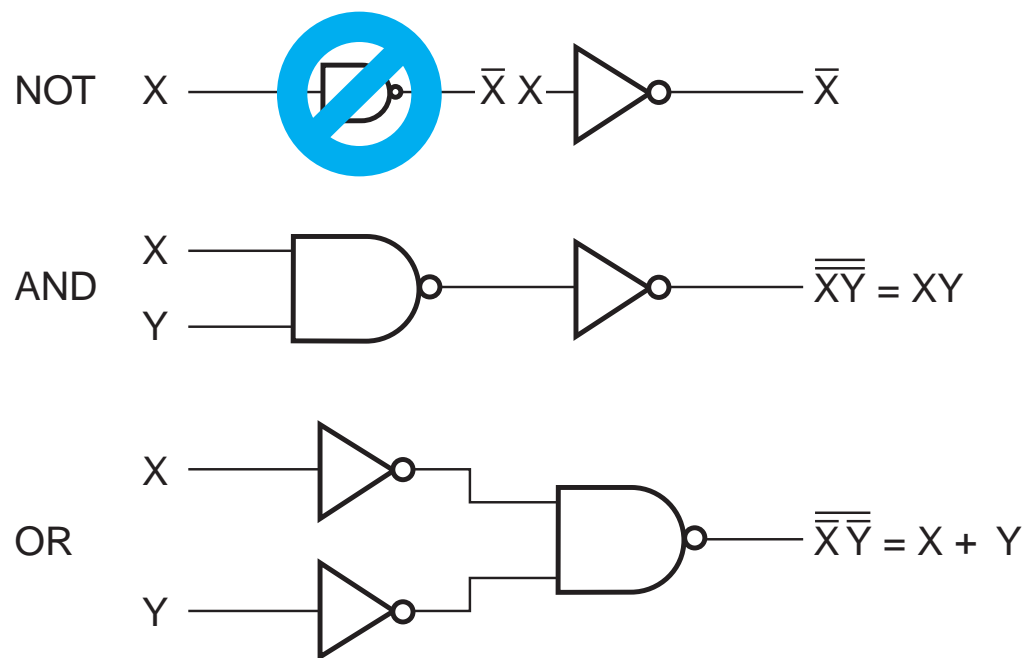
### Four-Variable Map: Flat and on a Torus to Show Adjacencies



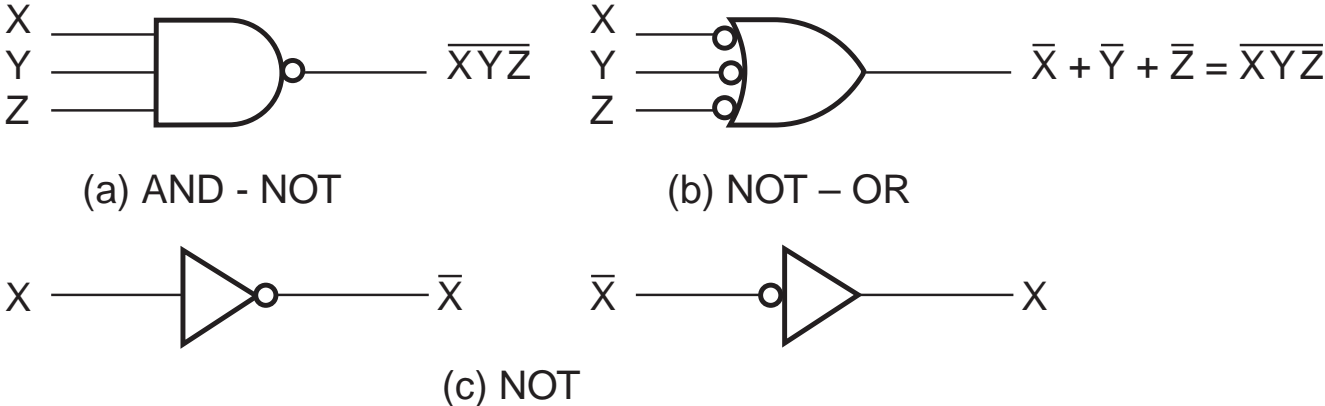
## Graphics Symbols

Name	Distinctive shape	Rectangular shape	Algebraic equation	Truth table															
AND			$F = XY$	<table border="1"> <thead> <tr> <th>X</th> <th>Y</th> <th>F</th> </tr> </thead> <tbody> <tr><td>0</td><td>0</td><td>0</td></tr> <tr><td>0</td><td>1</td><td>0</td></tr> <tr><td>1</td><td>0</td><td>0</td></tr> <tr><td>1</td><td>1</td><td>1</td></tr> </tbody> </table>	X	Y	F	0	0	0	0	1	0	1	0	0	1	1	1
X	Y	F																	
0	0	0																	
0	1	0																	
1	0	0																	
1	1	1																	
OR			$F = X + Y$	<table border="1"> <thead> <tr> <th>X</th> <th>Y</th> <th>F</th> </tr> </thead> <tbody> <tr><td>0</td><td>0</td><td>0</td></tr> <tr><td>0</td><td>1</td><td>1</td></tr> <tr><td>1</td><td>0</td><td>1</td></tr> <tr><td>1</td><td>1</td><td>1</td></tr> </tbody> </table>	X	Y	F	0	0	0	0	1	1	1	0	1	1	1	1
X	Y	F																	
0	0	0																	
0	1	1																	
1	0	1																	
1	1	1																	
NOT (inverter)			$F = \bar{X}$	<table border="1"> <thead> <tr> <th>X</th> <th>F</th> </tr> </thead> <tbody> <tr><td>0</td><td>1</td></tr> <tr><td>1</td><td>0</td></tr> </tbody> </table>	X	F	0	1	1	0									
X	F																		
0	1																		
1	0																		
Buffer			$F = X$	<table border="1"> <thead> <tr> <th>X</th> <th>F</th> </tr> </thead> <tbody> <tr><td>0</td><td>0</td></tr> <tr><td>1</td><td>1</td></tr> </tbody> </table>	X	F	0	0	1	1									
X	F																		
0	0																		
1	1																		
NAND			$F = \overline{X \cdot Y}$	<table border="1"> <thead> <tr> <th>X</th> <th>Y</th> <th>F</th> </tr> </thead> <tbody> <tr><td>0</td><td>0</td><td>1</td></tr> <tr><td>0</td><td>1</td><td>1</td></tr> <tr><td>1</td><td>0</td><td>1</td></tr> <tr><td>1</td><td>1</td><td>0</td></tr> </tbody> </table>	X	Y	F	0	0	1	0	1	1	1	0	1	1	1	0
X	Y	F																	
0	0	1																	
0	1	1																	
1	0	1																	
1	1	0																	
NOR			$F = \overline{X + Y}$	<table border="1"> <thead> <tr> <th>X</th> <th>Y</th> <th>F</th> </tr> </thead> <tbody> <tr><td>0</td><td>0</td><td>1</td></tr> <tr><td>0</td><td>1</td><td>0</td></tr> <tr><td>1</td><td>0</td><td>0</td></tr> <tr><td>1</td><td>1</td><td>0</td></tr> </tbody> </table>	X	Y	F	0	0	1	0	1	0	1	0	0	1	1	0
X	Y	F																	
0	0	1																	
0	1	0																	
1	0	0																	
1	1	0																	
Exclusive-OR (XOR)			$F = X\bar{Y} + \bar{X}Y$ $= X \oplus Y$	<table border="1"> <thead> <tr> <th>X</th> <th>Y</th> <th>F</th> </tr> </thead> <tbody> <tr><td>0</td><td>0</td><td>0</td></tr> <tr><td>0</td><td>1</td><td>1</td></tr> <tr><td>1</td><td>0</td><td>1</td></tr> <tr><td>1</td><td>1</td><td>0</td></tr> </tbody> </table>	X	Y	F	0	0	0	0	1	1	1	0	1	1	1	0
X	Y	F																	
0	0	0																	
0	1	1																	
1	0	1																	
1	1	0																	
Exclusive-NOR (XNOR)			$F = XY + \bar{X}\bar{Y}$ $= \overline{X \oplus Y}$	<table border="1"> <thead> <tr> <th>X</th> <th>Y</th> <th>F</th> </tr> </thead> <tbody> <tr><td>0</td><td>0</td><td>1</td></tr> <tr><td>0</td><td>1</td><td>0</td></tr> <tr><td>1</td><td>0</td><td>0</td></tr> <tr><td>1</td><td>1</td><td>1</td></tr> </tbody> </table>	X	Y	F	0	0	1	0	1	0	1	0	0	1	1	1
X	Y	F																	
0	0	1																	
0	1	0																	
1	0	0																	
1	1	1																	

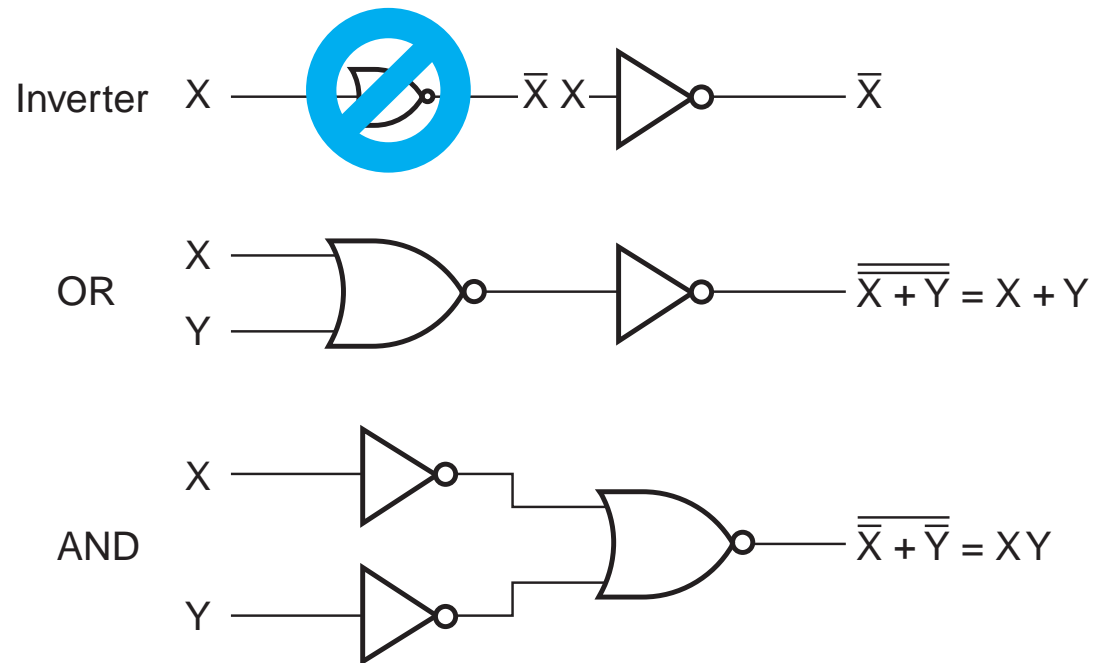
## Logical Operations with NAND Gates



Alternative Graphics Symbols for NAND and NOT Gates

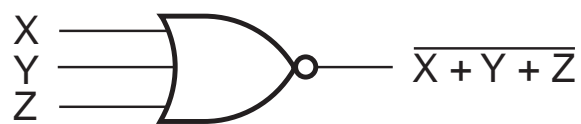


### Logic Operations with NOR Gates

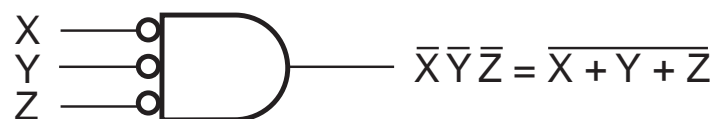




## Two Graphic Symbols for NOR Gate



(a) OR – NOT

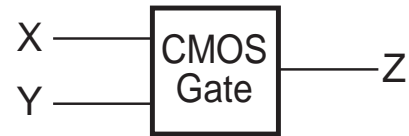


(b) NOT – AND

## Demonstration of Positive and Negative Logic

X	Y	Z
L	L	L
L	H	L
H	L	L
H	H	H

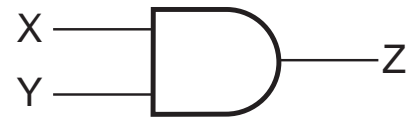
(a) Truth table with H and L



(b) Gate block diagram

X	Y	Z
0	0	0
0	1	0
1	0	0
1	1	1

(c) Truth table for positive logic



(d) Positive-logic AND gate

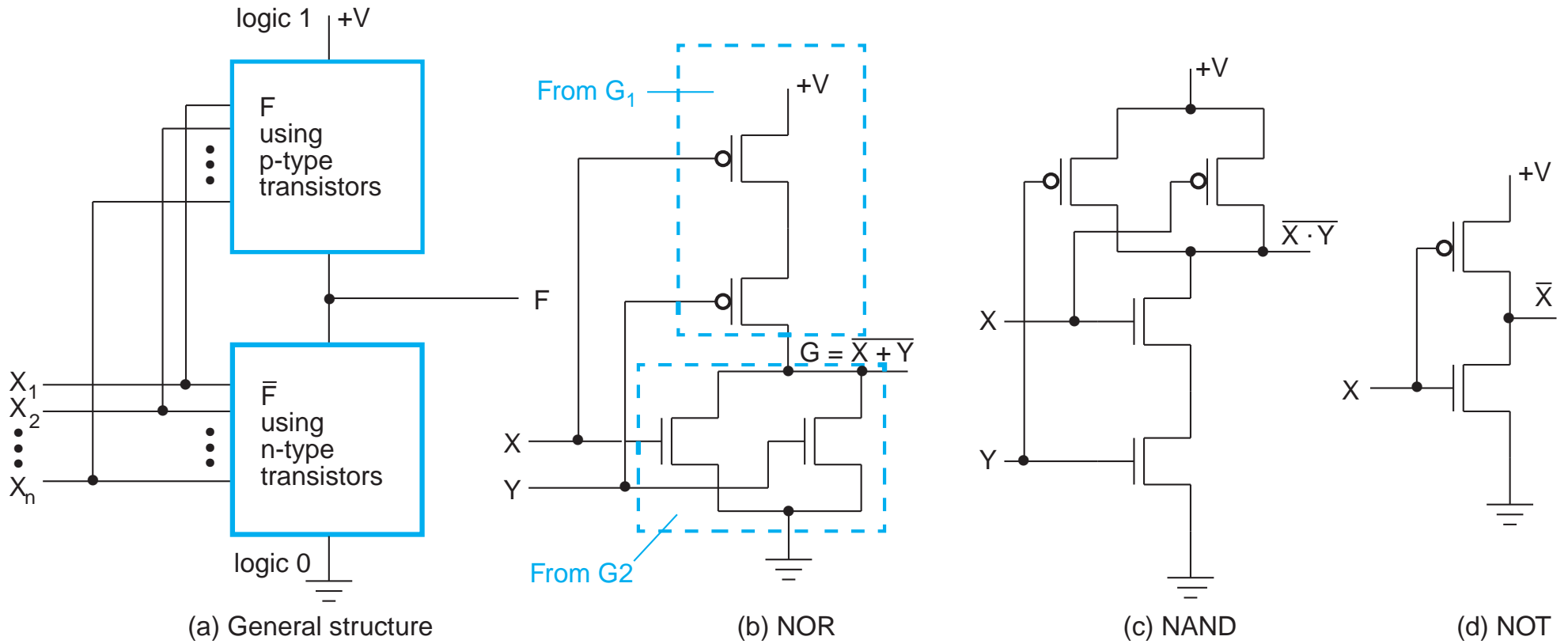
X	Y	Z
1	1	1
1	0	1
0	1	1
0	0	0

(e) Truth table for negative logic

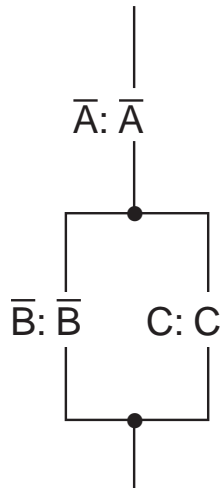


(f) Negative-logic OR gate

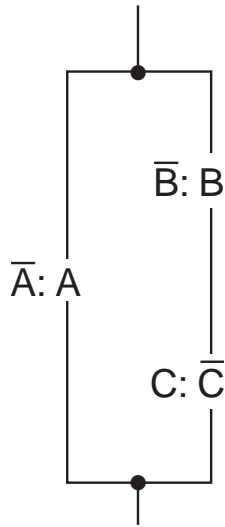
# Fully Complementary CMOS Gate Structure and Examples



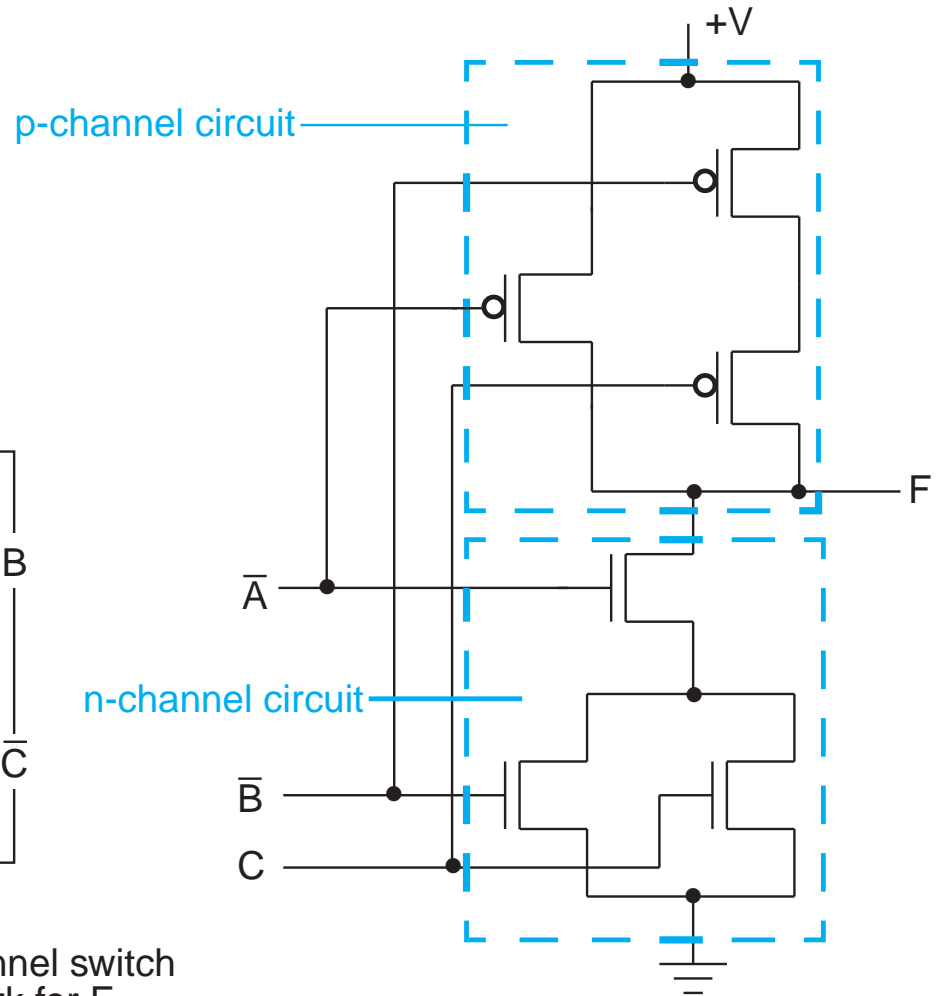
Networks and Circuit for Example 2-10



(a) n-channel switch network for  $F$

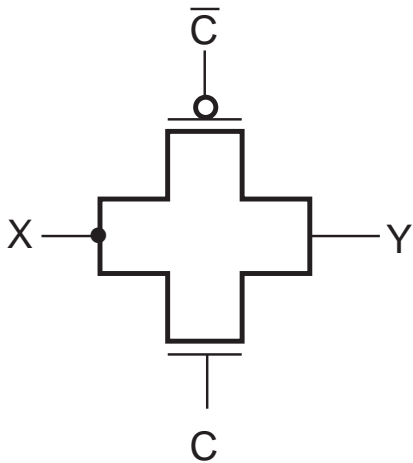


(b) p-channel switch network for  $F$

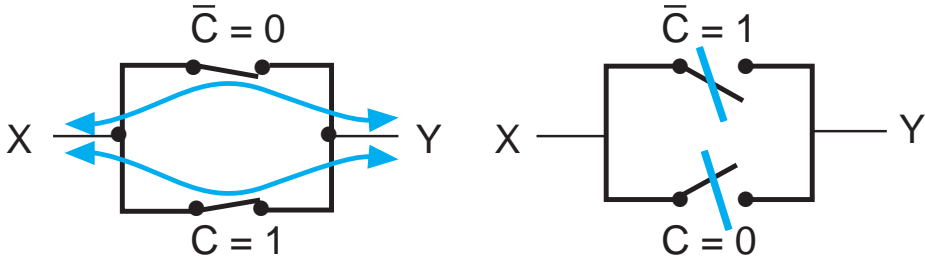


(c) transistor circuit for  $F$

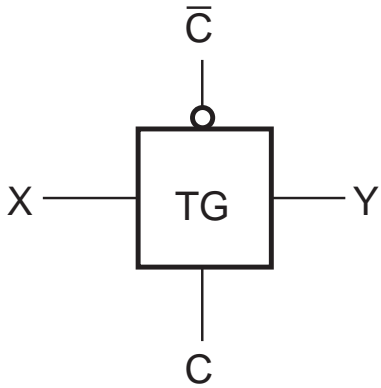
Transmission Gate (TG)



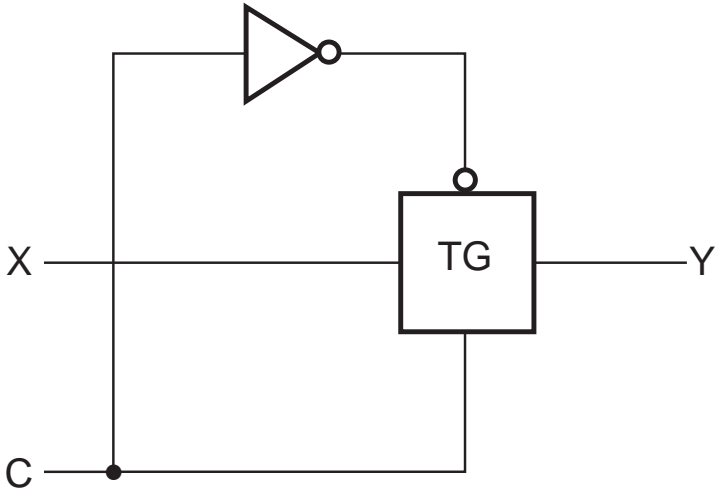
(a)



(b)

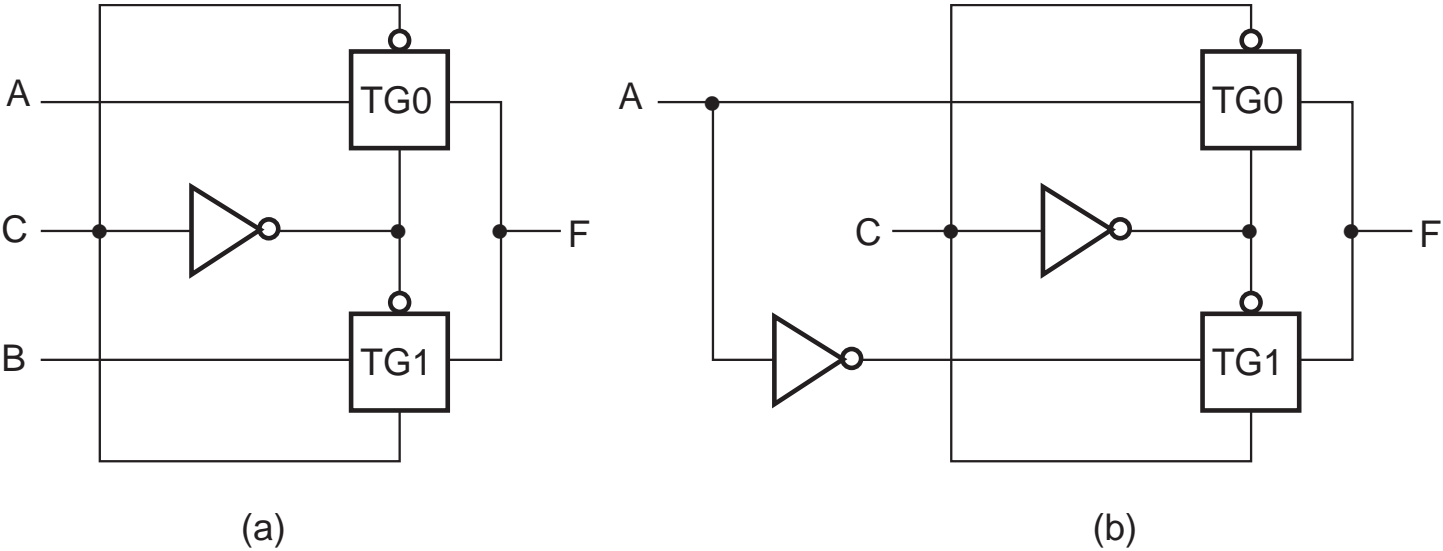


(c)



(d)

### Selector and Exclusive-OR Constructed with Transmission Gates



A	C	TG1	TG0	F
0	0	No path	Path	0
0	1	Path	No path	1
1	0	No path	Path	1
1	1	Path	No path	0

(c)